# **SOME USEFUL INFORMATIONS (you must understand these bounds first. They are the crux of Time bounding)**

**Note: Print this assignment and solve on it.**

**Arithmetic Sequence Size**1, 2, 3, 4, 5, 6, .... , N O(N)  
1, 3, 5, 7, .... , N N/2 i.e O(N)  
1, 4, 7, 10, .... , N N/3 i.e. O(N)  
**1, 1+k, 1+2k, 1+3k, 1+4k, 1+5k, .... , N N/k i.e. O(N) if k is a constant  
\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Arithmetic Series** Applications of **1+2+3+4+...+N =**

1+2+3+4+5+6+ ....N-3+ N-2+ N-1+ N O(N2)  
1+2+3+4+5+6+ ....(N/2-3)+ (N/2-2)+ (N/2-1)+ N/2 O(N2)  
1+2+3+4+5+6+ ....(N/3-3)+ (N/3-2)+ (N/3-1)+ N/3 O(N2)  
1+2+3+4+5+6+ .... + <= O(()2) O(N)  
1+2+3+4+5+6+ .... + N2 O(N4)  
1+2+3+4+5+6+ .... + N3 O(N6)  
**1+2+3+4+5+6+ .... +Nk <= O(NkxNk)**1+22+32+42+52+62+ .... + N2 O(N3)  
1+23+33+43+53+63+ .... + N3 O(N4)  
**1k+2k+3k+4k+5k+6k+ .... +Nk <= O(Nk+1)  
\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Geometric Sequence Size**N, N/2, N/4, N/8, N/24, N/25 , N/26, …8, 4 , 2 , 1 <=   
  
N, N/3, N/9, N/27, N/34, N/35 , N/36, … , 33, 9, 3 , 1 <=   
  
N, N/5, N/25, N/125, N/54, N/55 , N/56, …, 53, 52, 5 , 1 <=   
  
**N, N/k, N/k2, N/k3, N/k4, N/k5 , N/k6, … , k3, k2, k, 1 <=   
\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_** Application

O(log N)

**for(int i=1; i\*i<=N; i++)** Complexity of this loop is **O(**)  
 **Sum++;   
N x N = N2 for(int i=1; i\*i<=N\*N; i++)** Complexity of this loop is **O(**)  
 **Sum++;**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  
GEOMETRIC SERIES**

O(N)

**N < 1+2+4+8+16+32+... +N/4+N/2+N < 2N**N < 1+3+9+34+35+... +N/32+N/3+N < 2N  
  
N < 1+5+52+53+54+55+... +N/52+N/5+N < 2N

**Time complexity**

Single Loop

Increasing Order

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| I | 0 | 1 | 2 | 3 | … | N - 1 |
| Count | 1 | 2 | 3 | 4 | … | N | Time Complexity = O (N) |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| I | 1 | 2 | 3 | 4 | … | N |
| Count | 1 | 2 | 3 | 4 | … | N | Time Complexity = O (N) |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| I | 0 | 1 | 2 | 3 | … | N2 - 1 |
| Count | 1 | 2 | 3 | 4 | … | N2 | Time Complexity = O (N2 - 1) |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| I | 1 | 2 | 3 | 4 | … | N2 |
| Count | 1 | 2 | 3 | 4 | … | N2 | Time Complexity = O (N2) |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| I | 0 | 2 | 4 | 6 | … | 2k < N |
| Count | 1 | 2 | 3 | 4 | … | k = N/2 | Time Complexity = O (N/2) |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| I | 1 | 3 | 5 | 7 | … | 2k - 1 < N |
| Count | 1 | 2 | 3 | 4 | … | k = N/2 | Time Complexity = O (N/2) |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| I | 0 | 0 | 0 | 0 | … |  |  |
| Count | 1 | 2 | 3 | 4 | … |  | Time Complexity = Infinite |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| I | 1 | 2 | 4 | 8 | … | 2k <= N |  |
| Count | 1 | 2 | 3 | 4 | … | k = log2N | Time Complexity = O (log2N) |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| I | 0 | 1 | 2 | 3 | … | k < N |  |
| C (i) | 0\*0 | 1\*1 | 2\*2 | 3\*3 | … | k \* k < N |  |
| Count | 1 | 2 | 3 | 4 | … | √N | Time Complexity = O (√N) |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| I | 1 | 2 | 3 | 4 | … | N |  |
| C (i) | 1\*1 | 2\*2 | 3\*3 | 4\*4 | … | k \* k <=N |  |
| Count | 1 | 2 | 3 | 4 | … | √N | Time Complexity = O (√N) |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| I | 0 | 2 | 4 | 6 | … | 2k < N |  |
| C (i) | 0\*0 | 2\*2 | 4\*4 | 6\*6 | … | 2k \* 2k < N |  |
| Count | 1 | 2 | 3 | 4 | … | √N/2 | Time Complexity = O (√N/2) |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| I | 1 | 3 | 5 | 7 | … | 2k -1< N |  |
| C (i) | 1\*1 | 3\*3 | 5\*5 | 7\*7 | … | (2k-1) \* (2k-1) <= N |  |
| Count | 1 | 2 | 3 | 4 | … | √N/2 | Time Complexity = O (√N/2) |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| I | 1 | 2 | 4 | 8 | … | 2k <= N |  |
| C (i) | 1\*1 | 2\*2 | 4\*4 | 8\*8 | … | 2k \* 2k <= N |  |
| Count | 1 | 2 | 3 | 4 | … | log2√N | Time Complexity = O (log2√N) |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| I | 0 | 0 | 0 | 0 | … |  |  |
| C (i) | 0\*0 | 0\*0 | 0\*0 | 0\*0 | … |  |  |
| Count | 1 | 2 | 3 | 4 | … |  | Time Complexity = Infinite |

Single Loop

Decreasing Order

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| I | N | N - 1 | N – 2 | N - 3 | … | 1 |
| Count | 1 | 2 | 3 | 4 | … | N | Time Complexity = O (N) |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| I | N | N - 1 | N – 2 | N - 3 | … | 0 |
| Count | 1 | 2 | 3 | 4 | … | N - 1 | Time Complexity = O (N) |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| I | N2 | N2 - 1 | N2 - 2 | … | 1 |
| Count | 1 | 2 | 3 | … | N2 | Time Complexity = O (N\*N) |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| I | N2 | N2 - 1 | N2 - 2 | … | 0 |
| Count | 1 | 2 | 3 | … | N2 + 1 | Time Complexity = O (N\*N) |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| i | n | n - 2 | n – 4 | n - 6 | … | N > 2k |
| Count | 1 | 2 | 3 | 4 | … | k = N/2 | Time Complexity = O (N/2) |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| i | n | n - 2 | n – 4 | n - 6 | … | N >= 2k +1 |
| Count | 1 | 2 | 3 | 4 | … | k = N/2 | Time Complexity = O (N/2) |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| i | n | n/2 | n/4 | n/8 | … | N >= 2k |
| Count | 1 | 2 | 3 | 4 | … | k = log2N | Time Complexity = O (log2N ۤ + 1) |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| i | n | n/2 | n/4 | n/8 | … | N >= 2k +1 |
| Count | 1 | 2 | 3 | 4 | … | k = log2N | Time Complexity = O (log2N) |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| i | n | n/2 | n/4 | n/8 | … | N >= 2k |
| Count | 1 | 2 | 3 | 4 | … | k = log2N | Time Complexity = O (log2N) |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| i | n | n -1 | N - 2 | … | 1 |
| C (i) | n/n | (n – 1)/(n – 1) | (n -2) /(n – 2) | … | 1/1 |
| Count | 1 | 2 | 3 | … | N | Time Complexity = O (N) |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| i | n | n - 1 | N - 2 | … | 1 |
| C (i) | n/n | (n – 1)/(n – 1) | (n -2)/(n – 2) | … | 1 |
| Count | 1 | 2 | 3 | … | N | Time Complexity = O (N) |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| i | n | n – 2 | n - 4 | … | N > 2k |
| C (i) | n\*n | (n – 2)\*(n – 2) | (n - 4) \*(n – 4) | … | N > (2k \* 2k) |
| Count | 1 | 2 | 3 | … | N/2 | Time Complexity = O (N/2) |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| i | N | n - 2 | n - 4 | … | N > 2k |
| C (i) | n\*n | (n – 2)\*(n – 2) | (n - 4) \*(n – 4) | … | N > (2k \* 2k) |
| Count | 1 | 2 | 3 | … | N/2 | Time Complexity = O(N/2) |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| i | N | n/2 | n/4 | n/8 | … | N > 2k |
| C (i) | n\*n | (n/2)\*(n/2) | (n/4)\*(n/4) | (n/8)\*(n/8) | … | N > (2k \* 2k) |
| Count | 1 | 2 | 3 | 4 | … | log2N | Time Complexity =  O (log2 N) |

Mixed loops

Increasing order

**29.**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| I | 0 | 1 | 2 | 3 | … | N - 1 |
| Count | 1 | 2 | 3 | 4 | … | N | Time Complexity = O (N) |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| J | 1 | 2 | 3 | 4 | … | N |
| Count | 1 | 2 | 3 | 4 | … | N | Time Complexity = O (N) |

**Total time=** N + N = 2N = O (N) while ignoring ‘2’ as constant.

**30.**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| I | 1 | 2 | 3 | 4 | … | N2 |
| Count | 1 | 2 | 3 | 4 | … | N2 | Time Complexity = O (N2) |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| J | 0 | 2 | 4 | 6 | … | 2j < N |
| Count | 1 | 2 | 3 | 4 | … | j = N/2 | Time Complexity = O (N/2) |

**Total time=** N2 + N/2 = O (N2) because it covers maximum number of steps.

**31**.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| I | 0 | 1 | 2 | 3 | … | N2 - 1 |
| Count | 1 | 2 | 3 | 4 | … | N2 | Time Complexity = O (N2) |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| J | 1 | 2 | 3 | 4 | … | N2 |
| Count | 1 | 2 | 3 | 4 | … | N2 | Time Complexity = O (N2) |

**Total time=** N2 + N2 = 2N2 = O (N2) because it covers maximum number of steps

**32**.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| I | 1 | 3 | 5 | 7 | … | 2k -1< N |  |
| C (i) | 1\*1 | 3\*3 | 5\*5 | 7\*7 | … | (2k-1) \* (2k-1) < N |  |
| Count | 1 | 2 | 3 | 4 | … | √N/2 | Time Complexity = O (√N/2) |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| J | 1 | 2 | 4 | 8 | … | 2m < N |  |
| C (j) | 1\*1 | 2\*2 | 4\*4 | 8\*8 | … | 2m \* 2m < N |  |
| Count | 1 | 2 | 3 | 4 | … | √ log2N | Time Complexity = O (log2√N) |

**Total time=** √N/2 + log2√N = O (√N/2) because it covers maximum number of steps

**33.**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| I | 0 | 1 | 2 | 3 | … | N - 1 |
| Count | 1 | 2 | 3 | 4 | … | N | Time Complexity = O (N) |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| J | 1 | 2 | 4 | 8 | … | 2m <= N |  |
| Count | 1 | 2 | 3 | 4 | … | m = log2N | Time Complexity = O (log2N) |

**Total time=** N + log2N = O (N) because it covers maximum number of steps

**34.**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| I | 0 | 1 | 2 | 3 | … | N – 1 |
| Count | 1 | 2 | 3 | 4 | … | Nth | Time Complexity = O (N) |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| J | 1 | 2 | 3 | 4 | … | N2 |
| Count | 1 | 2 | 3 | 4 | … | N2 | Time Complexity = O (N2) |

**Total time=** N + N2 = O (N2) because it covers maximum number of steps

**35.**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| I | 0 | 2 | 4 | 6 | … | 2k < N |
| Count | 1 | 2 | 3 | 4 | … | k = N/2 | Time Complexity = O (N/2) |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| J | 1 | 2 | 4 | 8 | … | 2m <= N |  |
| Count | 1 | 2 | 3 | 4 | … | m = log2N | Time Complexity = O (log2N) |

**Total time=** N/2 + log2N = O (N/2) because it covers maximum number of steps

**36.**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| I | 0 | 1 | 2 | 3 | … | k < N |  |
| C (i) | 0\*0 | 1\*1 | 2\*2 | 3\*3 | … | k \* k < N |  |
| Count | 1 | 2 | 3 | 4 | … | √N | Time Complexity = O (√N) |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| J | 1 | 2 | 4 | 8 | … | 2m <= N |  |
| Count | 1 | 2 | 3 | 4 | … | m = log2N | Time Complexity = O (log2N) |

**Total time=**  =√N + log2N = O (√N) because it covers maximum number of steps

**37.**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| I | 0 | 2 | 4 | 6 | … | 2k < N |
| Count | 1 | 2 | 3 | 4 | … | k = N/2 | Time Complexity = O (N/2) |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| J | 1 | 2 | 4 | 8 | … | 2m <= N |  |
| Count | 1 | 2 | 3 | 4 | … | m = log2N | Time Complexity = O (log2N) |

**Total time=** √N/2 + log2N = O (√N/2) because it covers maximum number of steps

**38.**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| I | 0 | 1 | 2 | 3 | … | N - 1 |
| Count | 1 | 2 | 3 | 4 | … | N | Time Complexity = O (N) |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| j | 0 | 1 | 2 | 3 | … | m < N |  |
| C (j) | 0\*0 | 1\*1 | 2\*2 | 3\*3 | … | m \* m < N |  |
| Count | 1 | 2 | 3 | 4 | … | √N | Time Complexity = O (√N) |

**Total time=** N+ √N = O (N) because it covers maximum number of steps.

Mixed Loops

Decreasing Order

**39**.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| I | N | N - 1 | N – 2 | N - 3 | … | 1 |
| Count | 1 | 2 | 3 | 4 | … | N | Time Complexity = O (N) |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| J | N | N - 1 | N – 2 | N - 3 | … | 1 |
| Count | 1 | 2 | 3 | 4 | … | N | Time Complexity = O (N) |

**Total time=** N+ N = O (N) because it covers maximum number of steps.

**40.**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| I | N | N - 1 | N - 2 | N - 3 | … | 0 |
| Count | 1 | 2 | 3 | 4 | … | N | Time Complexity = O (N) |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| J | N2 | N2 - 1 | N2 - 2 | … | 1 |
| Count | 1 | 2 | 3 | … | N2 | Time Complexity = O (N\*N) |

**Total time=** N + N2 = O (N2) because it covers maximum steps.

**41**.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| i | n | n - 2 | n - 4 | n - 6 | … | N > 2k |
| Count | 1 | 2 | 3 | 4 | … | k = N/2 | Time Complexity = O (N/2) |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| J | n | n/2 | n/4 | n/8 | … | N >= 2m |
| Count | 1 | 2 | 3 | 4 | … | m = log2N | Time Complexity = O (log2N) |

**Total time=** N/2 + log2N = O (N/2) because it covers maximum steps.

**42**.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| i | n | n/2 | n/4 | n/8 | … | N >= 2k |
| Count | 1 | 2 | 3 | 4 | … | k = log2N | Time Complexity = O (log2N) |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| J | n | n - 1 | N - 2 | … | 1 |
| C (j) | n/n | (n – 1)/(n – 1) | (n -2)/(n – 2) | … | 1 |
| Count | 1 | 2 | 3 | … | N | Time Complexity = O (N) |

**Total time=** log2N + N = O (N) because it covers maximum steps.

**43**.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| i | n | n - 2 | n – 4 | n - 6 | … | N > 2k |
| Count | 1 | 2 | 3 | 4 | … | k = N/2 | Time Complexity = O (N/2) |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| j | n | n/2 | n/4 | n/8 | … | N >= 2m |
| Count | 1 | 2 | 3 | 4 | … | m = log2N | Time Complexity = O (log2N) |

**Total time=** N/2 +log2N = O (N/2) because it covers maximum steps

**44.**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| I | N | N - 1 | N -2 | … | 1 |
| Count | 1 | 2 | 3 | … | N | Time Complexity = O (N) |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| J | n | n/2 | n/4 | n/8 | … | 2m <= N |  |
| Count | 1 | 2 | 3 | 4 | … | m = log2N | Time Complexity = O (log2N) |

**Total time=** N + log2N = O (N) because it covers maximum number of steps

**45.**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| I | N2 | N2 - 1 | N2 – 2 | … | 0 |
| Count | 1 | 2 | 3 | … | N2 + 1 | Time Complexity = O (N2) |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| J | n | n - 2 | n – 4 | n - 6 | … | N > 2m |
| Count | 1 | 2 | 3 | 4 | … | m = N/2 | Time Complexity = O (N/2) |

**Total time=** N2 + N/2 = O (N2) because it covers maximum number of steps

**46.**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| I | N2 | N2 - 1 | N2 - 2 | … | 0 |
| Count | 1 | 2 | 3 | … | N2 + 1 | Time Complexity = O (N2) |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| I | N2 | N2 - 1 | N2 - 2 | … | 0 |
| Count | 1 | 2 | 3 | … | N2 + 1 | Time Complexity = O (N2) |

**Total time=** N2 + N2 = O (N2) because it covers maximum number of steps

**47.**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| i | N | n – 2 | n - 4 | … | N > 2k |
| C (i) | n\*n | (n – 2)\*(n – 2) | (n - 4) \*(n – 4) | … | N > (2k \* 2k) |
| Count | 1 | 2 | 3 | … | N/2 | Time Complexity = O (N/2) |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| J | N | n/2 | n/4 | n/8 | … | N > 2m |
| C (j) | n\*n | (n/2)\*(n/2) | (n/4)\*(n/4) | (n/8)\*(n/8) | … | N > (2m \* 2m) |
| Count | 1 | 2 | 3 | 4 | … | og2N | Time Complexity =  O (log2 N) |

**Total time=** N/2 + log2 N = O (N/2) because it covers maximum number of steps

**48.**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| I | N | N - 1 | N – 2 | N - 3 | … | 0 |
| Count | 1 | 2 | 3 | 4 | … | N - 1 | Time Complexity = O (N) |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| J | n | n -1 | N - 2 | … | 1 |
| C (j) | n/n | (n – 1)/(n – 1) | (n -2)/(n – 2) | … | 1 |
| Count | 1 | 2 | 3 | … | N | Time Complexity = O (N) |

**Total time=** N +N = O (N) because it covers maximum number of steps

Nested Loops

Independent + Increasing Order

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| i | 0 | 1 | 2 | 3 | … | N - 1 |
| j | N times | N times | N times | N times | … | N times |

Outer Loop = O (n); Inner Loop = O (n)

Series = N times N = O (n2)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| i | 0 | 1 | 2 | … | N - 1 |
| j | N/2 times | N/2 times | N/2 times | … | N/2 times |

Outer Loop = O (n); Inner Loop = O (n/2)

Series = N/2 times N = O (n2/2)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| i | 0 | 2 | 4 | … | 2k < N |
| j | N times | N times | N times | … | N times |

Outer Loop = O (n/2); Inner Loop = O (n)

Series = N/2 times N = O (n2/2)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| i | 0 | 1 | 2 | … | k < N |
| j | Log2 n times | Log2 n times | Log2 n times | … | Log2 n times |

Outer Loop = O (n); Inner Loop = O (Log2 n)

Series = N times Log2 n = O (n Log2 n)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| i | 1 | 2 | 4 | … | 2k < N |
| j | N times | N times | N times | … | N times |

Outer Loop = O (Log2 n); Inner Loop = O (n)

Series = N times Log2 n = O (n Log2 n)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| i | 1 | 2 | 4 | … | 2k < N |
| j | Log2 n times | Log2 n times | Log2 n times | … | Log2 n times |

Outer Loop = O (Log2 n); Inner Loop = O (Log2 n)

Series = Log2 n times Log2 n = O (Log2 n x Log2 n) = O (Log2 n)2

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| i | 0 | 1 | 2 | … | k \* k < N |
| j | √N times | √N times | √N times | … | √N times |

Outer Loop = O (√N); Inner Loop = O (√N times)

Series = √N times √N times = O (n)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| i | 0 | 1 | 2 | … | k \* k < N |
| j | √N/2 times | √N/2 times | √N/2 times | … | √N/2 times |

Outer Loop = O (√N); Inner Loop = O (√N/2 times)

Series = √N times √N/2 times = O (n/2)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| i | 0 | 2 | 4 | … | 2k \* 2k < N |
| j | √N times | √N times | √N times | … | √N times |

Outer Loop = O (√N/2); Inner Loop = O (√N times)

Series = √N/2 times √N times = O (n/2)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| i | 1 | 2 | 4 | … | 2k \* 2k < N |
| j | √N times | √N times | √N times | … | √N times |

Outer Loop = O (log2√N); Inner Loop = O (√N times)

Series = log2√N times √N times = O (√N log2√N)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| i | 0 | 1 | 2 | … | k \* k < N |
| j | log2√N times | log2√N times | log2√N times | … | log2√N times |

Outer Loop = O (√N); Inner Loop = O (log2√N times)

Series = log2√N times √N times = O (√N log2√N)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| i | 1 | 2 | 4 | … | 2k \* 2k < N |
| j | log2√N times | log2√N times | log2√N times | … | log2√N times |

Outer Loop = O (√N); Inner Loop = O (log2√N times)

Series = log2√N times log2√N times = O (log2√N \* log2√N) or O (log2√N)2

Nested Loops

Independent + Decreasing Order

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| i | n | N - 1 | N - 2 | N - 3 | … | 1 |
| j | N times | N times | N times | N times | … | N times |

Outer Loop = O (n); Inner Loop = O (n)

Series = N times N = O (n2)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| i | n | N - 1 | N - 2 | … | 1 |
| j | N/2 times | N/2 times | N/2 times | … | N/2 times |

Outer Loop = O (n); Inner Loop = O (n/2)

Series = N/2 times N = O (n2/2)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| i | N | N - 2 | N - 4 | … | N > 2k |
| j | N times | N times | N times | … | N times |

Outer Loop = O (n/2); Inner Loop = O (n)

Series = N/2 times N = O (n2/2)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| i | N | N - 1 | N - 2 | … | N > k |
| j | Log2n times | Log2n times | Log2n times | … | Log2n times |

Outer Loop = O (n); Inner Loop = O (Log2n)

Series = N times Log2n = O (n Log2n /2)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| i | n | n/2 | n/4 | … | n/2k = n/2Log2n |
| j | N times | N times | N times | … | N times |

Outer Loop = O (Log2n); Inner Loop = O (n)

Series = N times Log2n = O (n Log2n)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| i | n | n/2 | n/4 | … | | n/2k = n/2Log2n |
| j | Log2n times | Log2n times | Log2n times | … | Log2n times | |

Outer Loop = O (Log2 n); Inner Loop = O (Log2 n)

Series = Log2 n times Log2 n = O (Log2 n Log2 n) = O (Log2 n)2

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| i | N | N - 1 | N - 2 | … | 1 |
| j | N – 1 times | N - 1 times | N – 1 times | … | N - 1 times |

Outer Loop = O (N); Inner Loop = O (n)

Series = N times (N – 1) = O (n2)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| i | n | N – 1 | N - 2 | … | 1 |
| j | Log2 n + 1 times | Log2 n + 1 times | Log2 n + 1 times | … | Log2 n + 1 times |

Outer Loop = O (N); Inner Loop = O (Log2 n + 1)

Series = N times (Log2 n + 1) times = O (n Log2 n)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| i | N | N - 2 | N - 4 | … | N – 2k , k = n/2 |
| j | N times | N times | N times | … | N times |

Outer Loop = O (n/2); Inner Loop = O (N)

Series = N/2 times N = O (n2)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| i | n | n/2 | n/4 | … | n/2k = n/2Log2n = 1 |
| j | N times | N times | N times | … | N times |

Outer Loop = O (log2 n); Inner Loop = O (N)

Series = log2 n times n = O (n log2 n)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| i | n | n - 1 | n - 2 | … | 1 |
| j | Log2 n times | log2 n times | log2 n times | … | log2 n times |

Outer Loop = O (N); Inner Loop = O (log2 n)

Series = n times log2 n = O (n log2 n)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| i | n | n/2 | n/4 | … | n/2k = n/2Log2n = 1 |
| j | log2 n times | log2 n times | log2 n times | … | log2 n times |

Outer Loop = O (log2 n); Inner Loop = O (log2 n)

Series = log2 n times log2 n = O (log2 n \* log2 n) or O (log2 n)2

Nested Loops

Dependent + Increasing Order

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| i | 0 | 1 | 2 | 3 | … | N – 1 |
| j | 1 time | 2 times | 3 times | 4 times | … | N times |

Outer Loop = O (n); Inner Loop = O (i)

Series = 1 + 2 + 3 + … + n = n(n + 1)/2 = O(n2)

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| i | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | … | N - 1 |
| j | 1 time | 1 time | 2 times | 2 times | 3 times | 3 times | 3 times | 3 times | 4 times |  | Log2 n times |

Outer Loop = O (n); Inner Loop = O (i/2)

Series = 1 + 2 + 4 + … + log2 n (geometric series) => 1(2log2 n) + 1 - 1)/1 = O(2n) = O(n)

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| i | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | … | N - 1 |
| j | - | - | 1 time | 2 times | 2 times | 3 times | 3 times | 3 times | 4 times |  | Log2 n times |

Outer Loop = O (n); Inner Loop = O (Log2 i)

Series = 1 + 2 + 3 + 4 + … + log2 n = log2 n (log2 n + 1)/2 = O (log2 n)2

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| i | 1 time | 3 times | 5 times | 7 times | 9 times | … | √n times |
| j | 1 time | 2 times | 3 times | 4 times | 5 times | … | (√n + 1) times |

Outer Loop = O (n); Inner Loop = O (√i)

**Note:** Only this thing is required to be discussed that how √n groups;

and the value (√n + 1) have been calculated.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| i | 4 times | 12 times | 20 times | 28 times | 36 times | … | √n/2 times |
| j | 1 time | 2 times | 3 times | 4 times | 5 times | … | (√n + 1)/2 times |

Outer Loop = O (n); Inner Loop = O (√i/2)

**Note:** Only this thing is required to be discussed that how √n/2 groups;

and the value (√n + 1)/2 have been calculated.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| i | 1 time | 3 times | 12 times | 48 times | 192 times | … | Log2 √n times |
| j | 0 time | 1 time | 2 times | 3 times | 4 times | … | (Log2 √n) + 1 times |

Outer Loop = O (n); Inner Loop = O (Log2 i)

**Note:** Only this thing is required to be discussed that how Log2 √n groups;

and the value (Log2 √n + 1) have been calculated.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| i | 0 | 2 | 4 | 6 | … | 2k < N and k = n/2 |
| j | 1 time | 3 times | 5 times | 7 times | … | n/4 times |

Outer Loop = O (N/2); Inner Loop = O (i)

Series = 1 + 3 + 5 + … + n/4 = (n/4)2 =O (n2)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| i | 0 | 2 | 4 | 6 | … | 2k < N and k = n/2 |
| j | 1 time | 2 times | 3 times | 4 times | … | n/2 times |

Outer Loop = O (N/2); Inner Loop = O (i)

Series = 1 + 2 + 3 + … + n/2 = n/2 (n/2 + 1)/2 = O (n/2)2 = O (n2)

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| i | 0 | 1 time | 2 times | 3 times | 4 times | … | 2k < N and k = n/2 times |
| j | - | 1 time | 2 times | 3 times | 4 times | … | n/2 times |

Outer Loop = O (N/2); Inner Loop = O (log2 i)

Series = 1 + 2 + 3 + … + n/2 = n/2 (n/2 + 1)/2 = (n/2)2 = O (n2)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| i | 2 time | 6 times | 10 times | 14 times | … | Solve yourself |
| j | 1 time | 2 times | 3 times | 4 times | … | Solve yourself |

Outer Loop = O (logN/2); Inner Loop = O (√i)

**Note:** Only this thing is required to be discussed that how groups;

and the respective value have been calculated.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| i | 0 | 1 time | 2 times | 3 times | 4 times | … | Solve yourself |
| j | - | 1 time | 2 times | 3 times | 4 times | … | Solve yourself |

Outer Loop = O (N/2); Inner Loop = O (√i/2 times)

**Note:** Only this thing is required to be discussed that how groups;

and the respective value have been calculated.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| i | 0 | 1 time | 2 times | 3 times | 4 times | … | Solve yourself |
| j | - | 1 time | 2 times | 3 times | 4 times | … | Solve yourself |

Outer Loop = O (√N); Inner Loop = O (log2√N times)

**Note:** Only this thing is required to be discussed that how groups;

and the respective value have been calculated.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| i | 1 | 2 | 4 | 8 | … | 2k <= n |
| j | 1 time | 2 times | 4 times | 8 times |  | Log2 n times |

Outer Loop = O (log2 n); Inner Loop = O (i)

Series = 1 + 2 + 4 + … + log2 n (geometric series)

=> 1 (2log2 n) + 1 - 1) / 1

=> O (2n) = O (n)

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| i | 1 | 2 | 4 | 8 | 16 | 32 | … | 2k <= n => k = Log2 n |
| j | 1 time | 1 time | 2 times | 4 times | 8 times | 16 times |  | 2Log2 n / 2 = n/2 times |

Outer Loop = O (log2 n); Inner Loop = O (i)

Series = 1 + 1 + 2 + 4 + … + 2Log2 n / 2 (geometric series)

=> 1 (2log2 n) + 1 - 1) / 1

=> O (2n +1) = O (n)

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| i | 1 | 2 | 4 | 8 | 16 | 32 | … | 2k <= n => k = Log2 n |
| j | 0 time | 1 time | 2 times | 4 times | 8 times | 16 times |  | 2Log2 n / 2 = n/2 times |

Outer Loop = O (log2 n); Inner Loop = O (i/2)

Series = 1 + 2 + 4 + … + 2Log2 n / 2 (geometric series)

=> 1 (2log2 n) + 1 - 1) / 1

=> O (2n) = O (n)

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| i | 1 | 2 | 4 | 8 | 16 | … | 2k <= n => k = Log2 n |
| j | 0 time | 1 time | 2 times | 3 times | 4 times |  | Log2 n times |

Outer Loop = O (log2 n); Inner Loop = O (log2 i)

Series = 1 + 2 + 3 + 4 + … + Log2n (arithmetic series)

=> Log2n (Log2n + 1) / 2 = O (Log2n)2